

MAS 117

Introduction to Algebra

Assignment 4

For handing in on 7 February 2007

*Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the **RED** post-box on the **BASEMENT** of the Maths building before 16:00 on Wednesday.*

1 (30 marks) Which of the following relations R on sets X are (i) reflexive, (ii) symmetric, (iii) transitive? Give proofs or counterexamples.

- (a) $X = \mathbb{N}$, $(a, b) \in R$ if and only if a divides b .
- (b) $X = \mathbb{R}$, $(a, b) \in R$ if and only if $a > b$.
- (c) $X = \mathbb{C}$, $(a, b) \in R$ if and only if $|a| = |b|$.
- (d) $X = \mathbb{Z}$, $(a, b) \in R$ if and only if $a + b$ is a multiple of 5.
- (e) X is the set of countries of Europe, $(a, b) \in R$ if and only if a and b have a common border.

2 (10 marks) In the preceding question, in each case where R is an equivalence relation, describe the equivalence classes of R .

3 (20 marks) One of the following is not a function; which one, and why? In the other cases, is the function one-to-one? is it onto?

- (a) The function $F : \mathbb{C} \rightarrow \mathbb{R}$ given by $F(z) = \arg(z)$.
- (b) The function $F : \mathbb{R} \rightarrow \mathbb{R}$ given by $F(x) = x^3 - x$.
- (c) The function $F : \mathbb{R} \rightarrow \mathbb{R}$ given by $F(x) = e^x$.

4 (20 marks) Consider the following argument:

False proposition If a relation is symmetric and transitive then it is reflexive.

Proof Let R be a symmetric and transitive relation. Take $(x, y) \in R$. Then $(y, x) \in R$ (since R is symmetric), and so $(x, x) \in R$ (since R is transitive; put $z = x$ in the transitive law). So R is reflexive.

- (a) Say what is wrong with this argument.
- (b) Give a counterexample to the false proposition.

5 (20 marks)

- (a) How many functions are there from $X = \{1, 2, 3\}$ to $Y = \{1, 2\}$? How many of them are one-to-one? How many are onto?
- (b) How many functions are there from $X = \{1, 2\}$ to $Y = \{1, 2, 3\}$? How many of them are one-to-one? How many are onto?

6 Prize question A cash prize is offered for the best solution received to this question. Do not hand this in with the rest of your coursework; give your solutions to me.

Given two positive integers m and n , how many functions are there from $\{1, \dots, m\}$ to $\{1, \dots, n\}$? How many of them are one-to-one? How many are onto?